DRAGON Technical Note

# The Gas Target Density Profile

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## 1 Introduction

It is important to know the density profile $\rho(z)$ of the DRAGON gas target. The density profile determines the rate of energy loss by a beam as it passes through the target, thus providing the link between the energy-dependent quantities such as the capture cross section and the position-dependent factors such as recoil-ion acceptance and gamma detection efficiency.

The purpose of this note is to combine the pressure and total-thickness data with 1.5-mm vs 6/8-mm apertures and empirical gas-flow formulae, to constrain the density profile $\rho(z)$.

## 2 Pressure and Energy-Loss Data

In March 2002 the standard gas cell apertures of 6 mm and 8 mm were replaced by apertures of 1.5 mm for tests. (Hardware Logbook #2, page 27). The intention was to create a well-understood profile because gas flows would be low and high-pressure to low-pressure transitions would take place within a few mm. These were followed by measurements of beam energy loss versus pressure for the two configurations.

Table 1: Pressures in the outlet from the Roots blowers (CMTRIN) and inlet to the Roots blowers (CM1) in Torr, and the compression ratio $k_0$ for 1.5 mm apertures.

| CMTRIN | CM1 | $k_0$ |
|---|---|---|
| 0.562 | 0.027 | – |
| 10.5 | 0.029 | (5K) |
| 20.2 | 0.034 | 2800 |
| 33.0 | 0.049 | 1475 |
| 44.8 | 0.073 | 960 |

## 2.1 Pressure ratios

The first set of measurements were made with gas in the recirculation loop but no gas flow into the cell. The comparison of the pressure at the high-pressure outlet of the chain of Roots blowers (CMTRIN) versus that at the low-pressure inlet side (CM1) measures the maximum compression ratio, $k_0$, of the 3-stage Roots blower system. Table 1 lists the results. The compression ratio at 44.8 Torr is in reasonable agreement with what's expected for $H_2$ gas and 3 stages of blowers. The variation of compression ratio with outlet pressure is in the direction expected from published curves.

Assume that when there is gas flow through the cell, the back-flow through the blowers contributes to CM1 an amount equal to CMTRIN divided by $k_0$. A final correction to CM1 is to subtract the baseline pressure reading 0.027 Torr to give $CM1_{flow}$, the pressure rise associated with gas flow from the cell. Results from the March test and April/02 stable-beam runs (DRAGON Logbook #7, page 7) are presented in Table 2 Considering the small residual $CM1_{flow}$ for the 1.5 mm holes, there is a reasonably consistent ratio between it and the cell pressure.

Table 2: Pressures in the outlet from the Roots blowers (CMTRIN) and inlet to the Roots blowers (CM1) and in the gas cell (CMGC) in Torr, and the flow-related pressure $CM1_{flow}$ for 1.5 mm apertures.

| CMTRIN | CM1 | CMGC | $CM1_{flow}$/CMGC |
|---|---|---|---|
| 39.5 | 0.059 | 2.08 | 0.0000 |
| 39.5 | 0.066 | 4.50 | 0.0016 |
| 45.0 | 0.085 | 4.50 | 0.0025 |
| 8.6 | 0.037 | 4.60 | 0.0018 |
| 38.3 | 0.083 | 9.55 | 0.0025 |

A similar calculation can be performed for pressure data obtained with the standard apertures of 6 mm and 8 mm, Table 3. We can compute the expected ratio of pressures from the nominal pumping speed of two WSU2001 blowers in parallel (4000 $m^3$/hr = 1100 l/s) and the formula for $H_2$ flow (in Torr-l/s) through the apertures

$$q = 53.8d^2P$$

where d is the hole diameter in cm and P the cell pressure in Torr (from an expression given in U. Greife note to J. D'Auria, 30 Mar 98). Recalling that the holes are elliptical, on a 30° slant, the sum of the two $d^2$ terms will be (0.64 + 0.36)/cos(30) = 1.15 $cm^2$. For each 1 Torr of cell pressure the outflow is calculated to be 62.2 Torr-l/s, which, combined with nominal pumping speed 1100 l/s leads to an expected pressure ratio $CM1_{flow}$/CMGC = 0.057. This agrees very well with what we measure.

Table 3: Pressures in the outlet from the Roots blowers (CMTRIN) and inlet to the Roots blowers (CM1) and in the gas cell (CMGC) in Torr, and the flow-related pressure $CM1_{flow}$ for 6 mm and 8 mm apertures.

| CMTRIN | CM1 | CMGC | $CM1_{flow}$/CMGC |
|---|---|---|---|
| 12.0 | 0.150 | 2.30 | 0.053 |
| 14.6 | 0.190 | 3.00 | 0.053 |
| 19.5 | 0.271 | 4.40 | 0.054 |
| 42.0 | 0.374 | 5.00 | 0.055 |
| 27.2 | 0.408 | 6.60 | 0.056 |
| 34.0 | 0.547 | 8.80 | 0.056 |
| 37.1 | 0.615 | 9.77 | 0.057 |

### 2.2 Energy loss ratio

Measurements of beam energy loss versus cell pressure were carried out for both the 1.5/1.5 mm and 6/8 mm configurations. The ratio of slopes gives the ratio of effective lengths. With the assumption of an effective length 110±2 mm for the 1.5 mm case, the effective length with the standard 6/8 mm apertures is 123±5 mm, an increase of 12%.

## 3 A Model for the Pressure Profile

The density profile is described as the sum of two parts, each part described by a shape factor multiplied by a scaling pressure. The free parameters will be defined so as to be relatively insensitive to changes in aperture size and to be constrained by the gas pressure data shown above.

- An "outer" zone consisting of the top end of the vacuum box (where the beam passes) and the closest pumping tubes. It is characterized by a pressure in the box (including the inside of the cell) which is a fraction f of the central cell pressure and which falls linearly to zero along the pumping tubes. The effective length is the length of the box plus one-half the summed lengths of the two pumping tubes: $L_{eff}^{outer}$ = 310 mm.

- An "inner" zone consisting of the gas cell and whatever distance outside it is required to get down to the baseline "outer" pressure. At z=0 its pressure is (1-f) of the central pressure. The z dependence is taken to be a Fermi function

$$(1 + exp(\zeta))^{-1}$$

where $\zeta = (\mid z \mid - c_{\pm})/(b_{\pm})$. The width of the region of rapid pressure change is controlled by the parameter b, which will be different for the different apertures at positive and negative z. The position of the rapid falloff in pressure is controlled by the parameter c which, again, may be slightly different for the two apertures.

As discussed in the Appendix, the contribution of each pumping tube to $L_{eff}^{outer}$ is approximately 0.5 of its 152-mm physical length. More than 50% of this gas will be in the 50 mm section that is closest to the vacuum box. At a guess, the uncertainty in the coefficient 0.5 is ±0.05 which is an uncertainty of 15 mm in the nominal value of 310 mm for $L_{eff}^{outer}$.

The effective length of the inner region is simply the sum of the parameters $c_+$ and $c_-$. (It is a useful property of the Fermi function that this remains true also if $\zeta$ is replaced by a function which is odd in $\zeta$.) The value of c need not coincide exactly with the position of the aperture, although we expect them to be close and any difference to scale with the aperture size $d_+$ or $d_-$. Allow for this possibility by defining $\delta$ such that

$$c_+ + c_- = L_{phys}^{inner} + \delta(d_+ + d_-)$$

. Now the overall effective length of the gas target can be written as

$$L_{eff}^{total} = (1-f)[L_{phys}^{inner} + \delta(d_+ + d_-)] + fL_{eff}^{outer}$$

.

It is helpful to constrain the relative "outer" pressure f by the CM1 pressure data. Introduce a parameter C to describe the conductance of the (very crowded) vacuum box between the volumes at beam height and the volume that contains gauge CM1. By definition, the pressure difference ΔP between the top of the box and CM1 is related to C and the gas flow q by ΔP=q/C. For a central cell pressure CMGC, blower outlet pressure CMTRIN, compression ratio $k_0$, and blower pumping speed S, set

$$\phi = 53.8(d_+^2 + d_-^2)(1/S + 1/C)$$

. This is the pressure ratio CM1/CMGC ignoring the reduction of flow out the apertures due to back-pressure. Inclusion of this reduction factor gives

$$f = [\phi + CMTRIN/(k_0 \cdot CMGC)]/[1+\phi]$$

(If the apertures are elliptical, d must be replaced by the geometric mean of the major and minor axes.)

f is constrained by the requirement C>0. As discussed in the Appendix, the upper limit on C is very hard to set, and in any case is pressure dependent. The "free" parameters of the model are the shift in effective length $\delta$, the width of the zone of rapid pressure drop b, and the box conductance C: these cover our inability to properly model the most complicated regions of gas flow in the DRAGON target. Our measurements of total effective length link the allowed values of $\delta$ and C but do not constrain b. Inserting into the effective length equation $L_{phys}^{inner}$=110, $L_{eff}^{outer}$=310, $L_{eff}^{total}$=123±5,

$$(0.935)[110 + 7\delta] + [0.056(1 + S/C) + 0.009]310 = 123 \pm 5$$

or

$$6.5\delta + 17(S/C) = 0 \pm 5mm$$

The estimated uncertainty in $L_{eff}^{outer}$ would add another ±1 mm to the right-hand side of the last equation.

# 4 Conclusions

1. Gas pressure at the inlet to the Roots blowers, CM1, scales in a reasonable way with changing outlet pressure CMTRIN and gas cell pressure CMGC.

2. Reasonable values are obtained for the maximum compression ratio of the 3-stage Roots blower chain.

3. The standard expression for gas flow through the cell apertures combined with the nominal pumping speed of the Roots blowers can reproduce the observed changes in CM1 to better than 10%.

4. A model of the gas density profile is presented. Its principal features are:

    - base pressure at beam height in the vacuum box which is constant at a pressure which is a fraction f of the central pressure of the gas cell

    - pressure which falls linearly from "f" at the high-pressure end to zero at the low-pressure end of the inner pumping tubes

    - Fermi function to describe the additional gas inside the cell and just outside the apertures.

    - fraction "f" described by plausible values of box conductance

- main dependences on aperture size given explicitly, so parameters of the model should be insensitive to choice of aperture.

5. Two of the model parameters, box conductance C and shift in the Fermi-function contribution to effective length $\delta$ are linked and constrained by the measurement of overall effective length. The result is that the base pressure at the entrance to the pumping tubes probably lies between 6.5% and 9% of the central cell pressure.

# APPENDICES

## A Pressure profile in a pumping tube

The two in-board pumping tubes are each 152 mm long divided into 3 equal lengths of stepped inner diameter. For the upstream tube the inner diameters are 8, 9, and 10 mm (smallest diameter nearest the the central vacuum box) and for the downstream tube 9, 10.4, and 11.2 mm.

The flow regime is transitional between high-pressure viscous flow and low-pressure molecular flow. The Knutsen flow equation (U. Greife, op cit.) for flow in a cylindrical tube of diameter d (cm), length L (cm) can be written in the form

$$q = 380d^4\frac{P_1^2 - P_2^2}{2L} + 46.1d^3 f(d \cdot \overline{P})\frac{P_1 - P_2}{L}$$

where f≈0.8 is a slowly varying function of $\overline{P}$=($P_1$+$P_2$)/2.

Setting $P_2$ to 0 in comparison to $P_1$, we see that the pressure and length appear as $P^2$/L in the first term, P/L in the second term. The first term dominates at high pressure in the viscous flow case, while the second term dominates at low pressure molecular flow.

Suppose molecular flow dominates. If we want to extend the length L without changing the flow q, the pressure must be increased in proportion to keep P/L unchanged. We deduce that the pressure profile will vary linearly with distance along the tube and the thickness of gas will be (1/2)$P_1$L. On the other hand, if the first term dominates the position will be a quadratic function of pressure and the thickness of gas will be (2/3)$P_1$L.

For a cylindrical tube we would expect the gas thickness to be somewhere between 1/2 and 2/3 of $P_1 \cdot L$. However, the steps in the internal diameter of the pumping tubes tend to reduce the coefficient, bringing the multiplier close to 0.5.

# B Transition region at the apertures

The gas flow out each aperture is directed along a symmetry axis at 30° to the beam axis. Inside the cell the gas flow is certainly subsonic, outside it is probably supersonic (F.M. White, *Fluid Mechanics*). It is not obvious how the pressure "deficiency" seen by the beam inside the cell compares with the pressure "surplus" it sees after it passes through the aperture. In other words, it may be that the $L_{eff}^{inner}$ is not exactly the distance between the apertures.

# C Conductance of the rectangular pumping box

The middle section of the vacuum box is filled by a maze of obstructions to gas flow: elastic monitor housings, fill-tube and its support block, pressure gauge tube, coax cables, and vacuum feed-throughs. It is not obvious how to adapt simple expressions such as the Knudsen equation for a cylindrical tube (given above) to this case: what is the analogue of tube diameter which enters as the 4th power? what is the length L?

To get a feeling for the problem, assume that the square of the cylindrical tube diameter (d) is to be equated to the "typical" open cross-sectional area seen by gas as it percolates down the box. Taking this typical area to be 16 cm by 1 cm, we arrive at d=4 cm. The encumbered zone is roughly a length L=10 cm. At an average pressure $\overline{P}$= 0.4 Torr, the Knudsen equation would give

$$C = q/(P_1 - P_2) = 5100 l/s$$

or S/C = 0.2. Of course, this was only a guess for demonstration purposes: we might have guessed at d=3 or d=5 and obtained S/C ratios which were larger by 3 or smaller by 2.5, respectively. However, it does demonstrate that the model is not order-of-magnitude wrong.