Gamma decay of the 7.65 MeV state in ^{27}Si

DRAGON Note

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1. Introduction

An unpublished 1989 proton capture experiment by Vogelaar[1] found the 7.65 MeV state of ^{27}Si to gamma decay 90% to a level at 4.45 MeV and 10% to the level at 2.91 MeV (Figure 1). A 2009 Gammasphere experiment by Lotay *et al.*[2] found a strong decay to the 4.45 MeV state and a weak decay to a state at 5.28 MeV (Figure 2).

Are these results compatible or do they show that different members of a closely spaced pair were excited in the two experiments? Vogelaar recalls[3] that the 90:10 solution was clearly preferred, but the data tapes are no longer readable so re-analysis "from scratch" is not possible. Given the considerable difficulties of mounting a proton capture experiment with ^{26g}Al either as target or as beam, it is unlikely that a re-measurement will occur in any near future.

This note reports results of a GEANT4 simulation of the Vogelaar experiment, with the goal of estimating the likelihood of obtaining Vogelaar's result under various assumptions about the true branching ratios of the 7.65 MeV level.

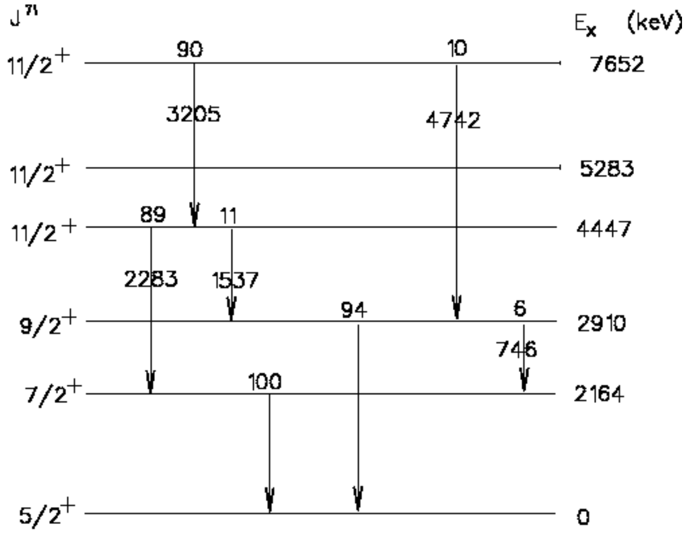

Figure 1. Gamma decay scheme proposed by Vogelaar[1] for the 7.65 MeV state of ^{27}Si.

2. GEANT4 simulation of the Vogelaar experiment

GEANT4[4] is a system for simulating the interaction of particles with matter, and finds applications in medical, particle and nuclear physics as well as other fields. In the

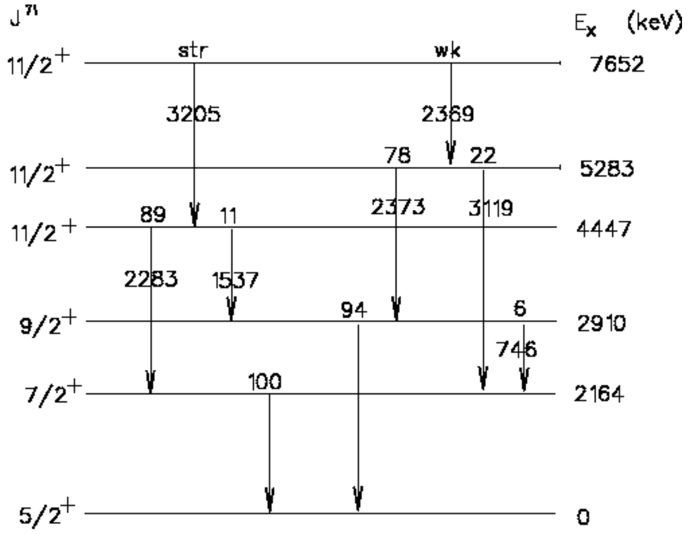

FIGURE 2. Gamma decay scheme proposed by Lotay *et al.*[2] for the 7.65 MeV state of ^{27}Si

present application we are concerned only with purely electromagnetic processes — the interaction of gamma rays, electrons and positrons with detectors and other material surrounding the target. The code for this study was developed by merging concepts illustrated in N01 and N03 of the Novice examples in the GEANT4 package.

2.1. **Detector construction.** The layout of the Vogelaar experiment is illustrated in Fig. 5.1 of Ref. [1]. Proton beam passed inside beampipe to a target where it stopped; the target backing was a Pt foil, 0.010" thick. Downstream of the target were two concentric pipes which conducted cooling water towards and away from the back of the Pt foil. A set of flanges connected the upstream and downstream pipes and also clamped the Pt foil in place. All this target assembly fit within a 1"x1" envelope.

The detectors were four large NaI crystals, each 6"×6"×10". They closely surrounded the target and were arranged in such a way as to enclose the long sides of a 1"×1"×10" volume. The NaIs in turn were surrounded by shielding against room and cosmic background, but neither the shielding nor the external background was included in the simulation.

The beampipes and flanges were assumed to be 304 stainless steel (70% Fe, 20% Cr, 10% Ni) and had the following dimensions: upstream beampipe 12 mm o.d., 10 mm i.d. extending to within 4 mm of the target plane; inner downstream water line 2 mm i.d., 4 mm o.d. extending to within 4 mm of the upstream face of the Pt foil ("z=0"); outer water line 6 mm i.d., 8 mm o.d. and same extent as the inner water pipe. The pair of flanges together were represented by a ring of 12 mm i.d., 20 mm o.d. and 8 mm thick, centred on the z axiz at z=0. The 0.010" Pt foil was placed with its upstream face at z=0. The connection between downstream flange and the outer water line was represented by a "washer" of 12 mm o.d. and 8 mm i.d., 1 mm thick.

Pipe thicknesses were plausible assumptions based on Fig. 5.1. No information was provided about the thickness of reflector and encapsulation of the NaI detectors. Because detector construction typically aims to minimize gamma-ray losses in the NaI containers, they were not included in the simulation.

2.2. **Generator of primary gamma rays.** Simulated gamma decay cascades from the 7.65 MeV state all were assumed to take place at the point x=y=z=0. Each event consisted of an initial gamma-ray decay to one of the levels of lower excitation energy, chosen by random selection according to branching ratios specified by the user. After selection of the initial transition, subsequent branches in the decay cascade were chosen at random according to branching ratios given in Fig. 3.8 of Ref. [1] (which have been reproduced in Figures 1 and 2).

After each of the 2 or 3 or 4 gamma-ray energies was chosen, the lab direction was picked at random from an isotropic distribution. This is justified by a separate study of angular correlations for a 3-gamma cascade $11/2 \to 11/2 \to 7/2 \to 5/2$. For the geometry of the CalTech experiment it is the correlation between azimuthal angles which largely determines the likelihood of 2 or 3 gamma rays being detected in the same NaI counter. Figure 3 shows the relative cross section as a function of difference in azimuthal angles for 1000 samplings of 3-gamma phase space. The proton capture was assumed to be s-wave and the multipole mixing ratio for the first step to be zero. No significant correlation is seen. The case of initial mixing ratio 0.35 and the case of p-wave capture with channel spin 9/2 similarly lacked obvious structure.

2.3. **Energy spectra.** For each event the simulated amount of energy deposited in each of the four NaI detectors was recorded. For each detector this deposited energy was smeared according to a Gaussian distribution with

$$\sigma = 0.03\sqrt{E_{dep}}$$

for energies in units of MeV. The four smeared energies were summed and the appropriate bin in a summed-energy histogram was incremented. If the summed energy lay between 7.2 and 7.75 MeV, the energy in each of the detectors was used to increment the appropriate channel in a "detector singles" histogram. The incrementing of the singles spectrum thus mirrored the procedure used in forming the upper histogram of Fig. 5.9 in Ref. [1]. The smearing factor was adjusted to provide splitting of peaks at 2.2–2.4 MeV which closely matched that of the simulation shown in the bottom panel of Fig. 5.9.

Histogram binning was chosen to be 50 keV/channel as a near match to that of Fig. 5.9 in the Vogelaar thesis.

3. Comparison of branching ratio models

This investigation will attempt to follow lines which might reasonably have been taken by Vogelaar[1], knowing that he relied on extensive EGS simulation and coincidence gating to identify the principal cascade through the 4.45 MeV state. The key

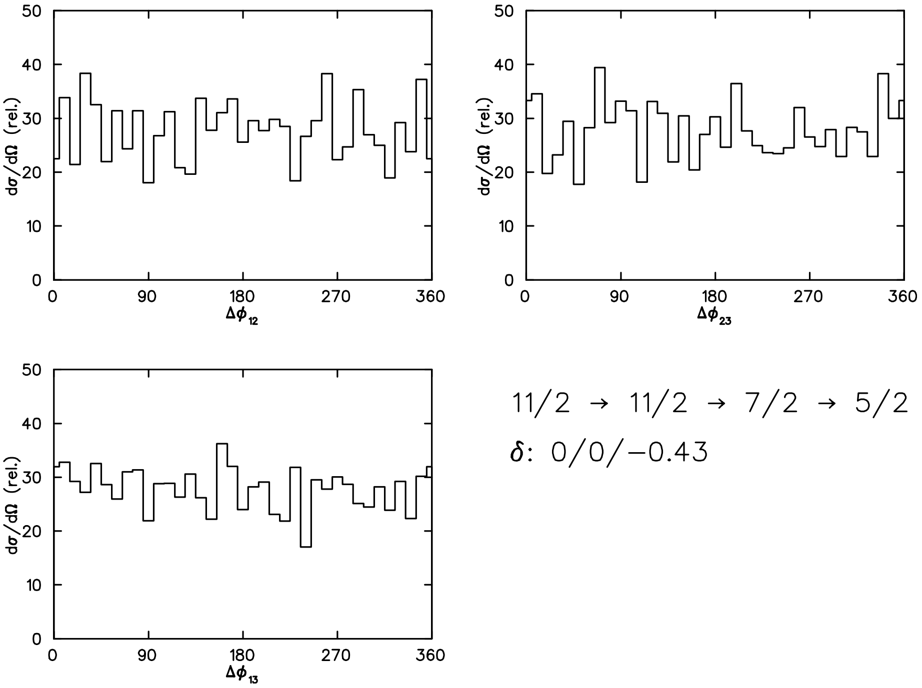

FIGURE 3. Correlations in azimuthal angles of gamma rays from a cascade through levels of spins 11/2, 11/2, 7/2,5/2.

information is[1]
"Using this branching information, the detector response was modeled with the EGS4 code. For the 196-keV resonance, one can see in Figure 5.9 that most of the spectrum can be accounted for by the triple cascade determined above; the line at 2.9 MeV requires another deexcitation mode. The final branching ratios are given in Figure 3.8." The procedure for calculating this branching ratio was not specified. As will be seen, the limited statistics, background and modest detector resolution militated against a fit with all possible branches as free parameters: error correlations cause large uncertainty in the best-fit value of individual branches.

3.1. **2-parameter decay fits.** We approach the problem by posing a specific question: can the "excess" in the 2.91 MeV peak be explained by a 2-branch decay — to the 4.45 MeV state and to just 1 other state? All states known (at the time of Vogelaar's analysis) to decay to the 2.91 MeV state should be considered as candidates. They are states at 4.45, 5.26, 5.28 and 5.55 MeV.

"Theoretical" lineshapes for each of the possible transitions from the 7.65 MeV state were generated by 20,000-event simulation runs where only one of the 5 transitions was allowed. Thus, in principle, the model was a perfect representation of (simulated!) reality. The 5 "pure" spectra are shown in Figs.4,5,6,7 and 8.

Three types of "experimental" spectra were generated by GEANT4: the "V" class had a 10% branch to the 2.91 MeV state, 90% to the 4.45 MeV state; the "L" class had 5.3% branching to the 5.28 MeV state, the balance to the 4.45 MeV level; the "7+7" class had 7% to the 5.28 MeV state, 7% to the 5.55 MeV state and 86% to the level

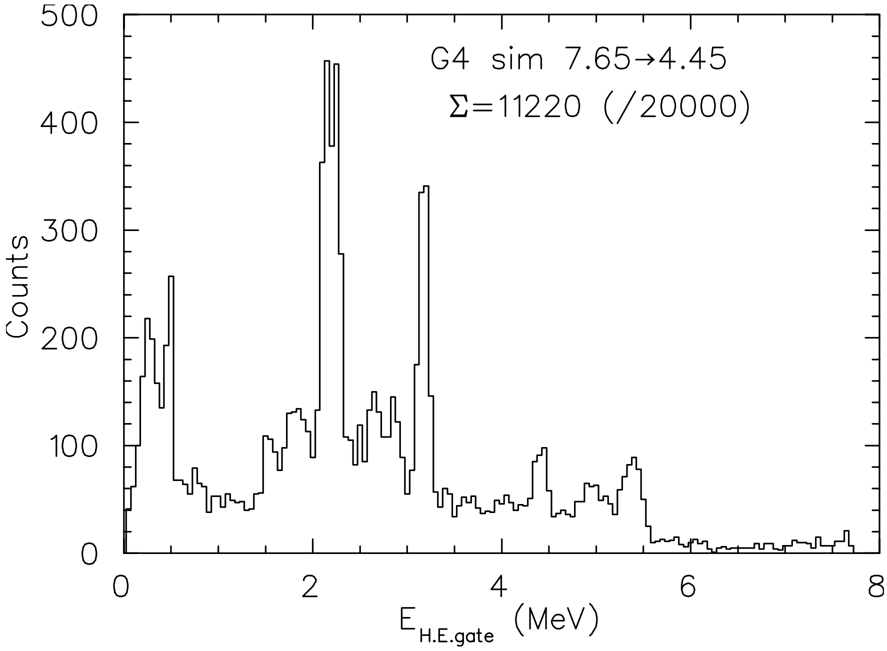

FIGURE 4. GEANT4 simulation of gamma cascades initiated by a 7.65→4.45 transition in ^{27}Si.

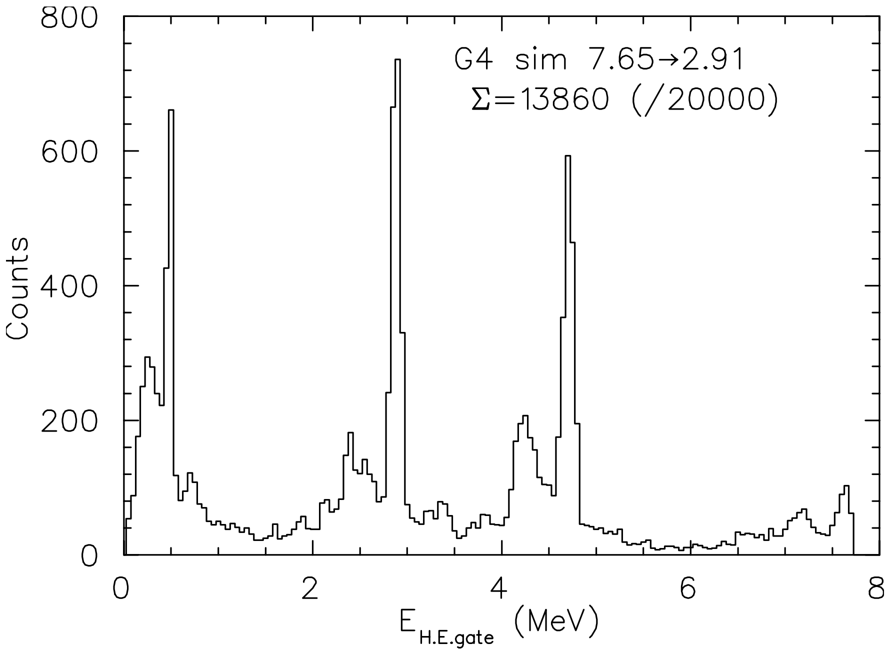

FIGURE 5. GEANT4 simulation of gamma cascades initiated by a 7.65→2.91 transition in ^{27}Si.

at 4.45 MeV. The "7+7" was selected because it populates the 2.91 MeV state with essentially the same strength as the "V" decay scheme.

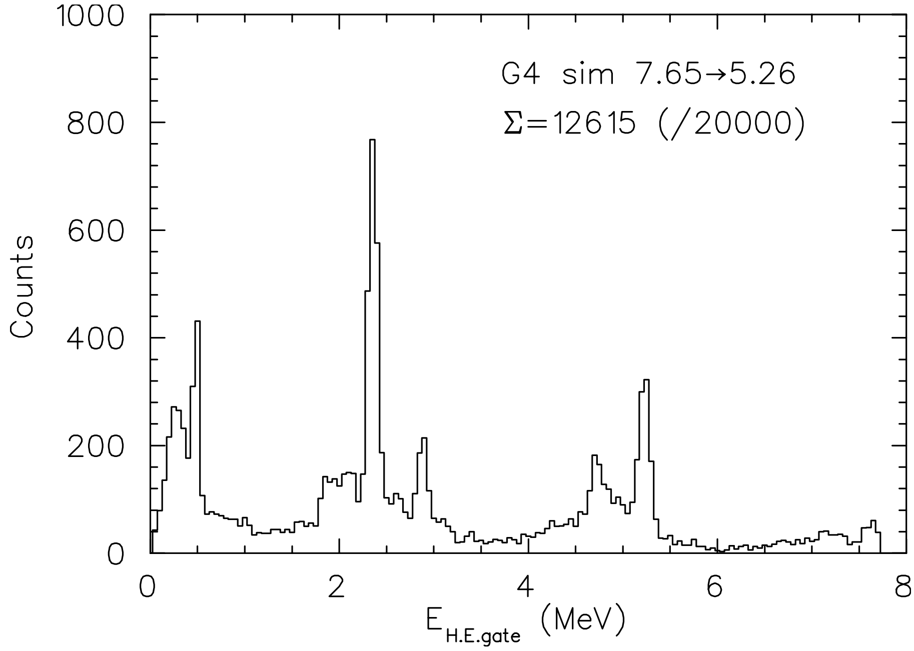

FIGURE 6. GEANT4 simulation of gamma cascades initiated by a 7.65→5.26 transition in ^{27}Si.

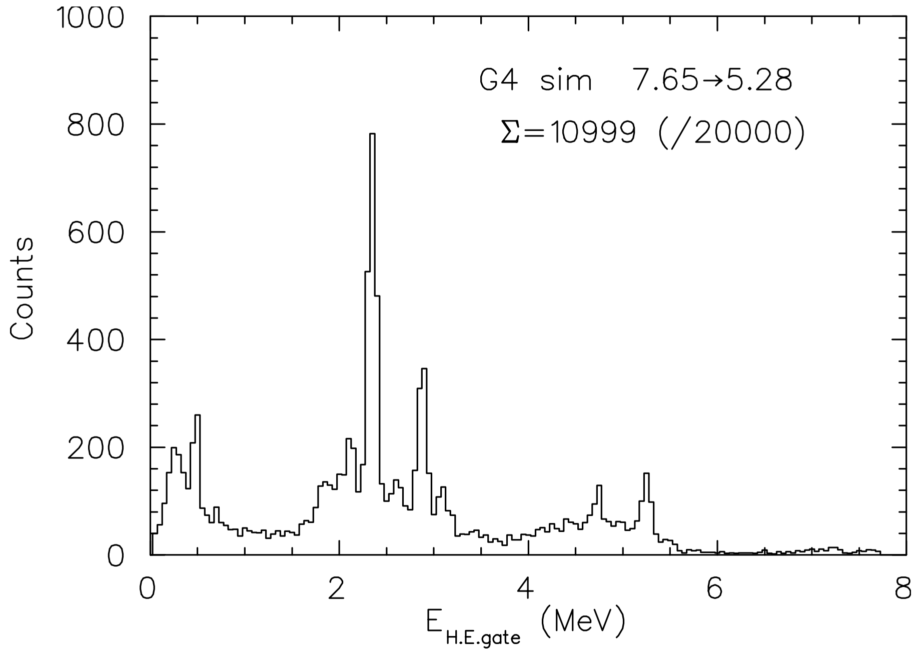

FIGURE 7. GEANT4 simulation of gamma cascades initiated by a 7.65→5.28 transition in ^{27}Si.

It was found empirically that simulated runs of 800 events produced peaks of about the height seen in Fig. 5.9 of Reference [1]. Due to the paucity of counts above 5.5 MeV

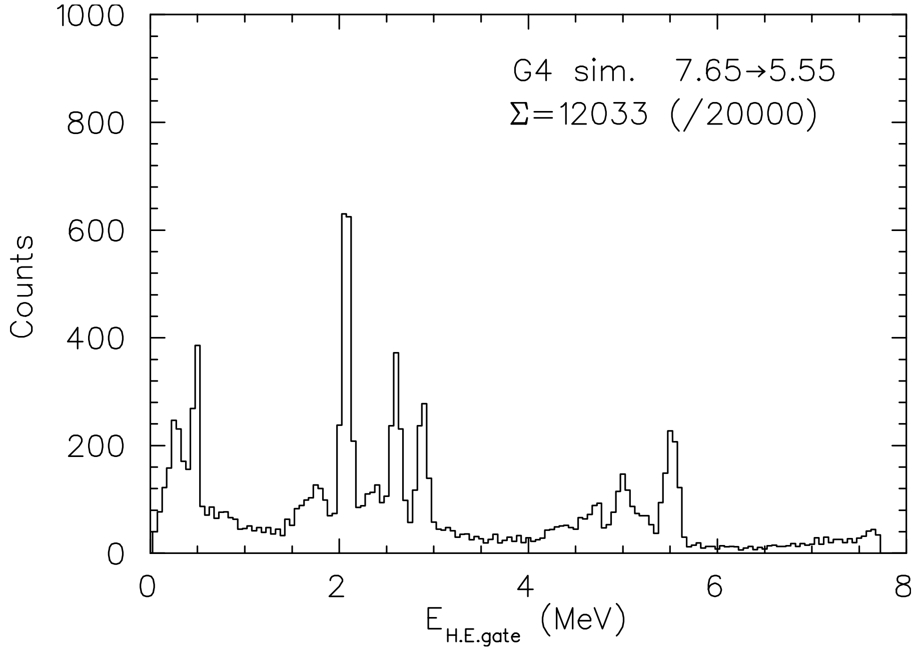

FIGURE 8. GEANT4 simulation of gamma cascades initiated by a 7.65→5.55 transition in ^{27}Si.

in the "data" spectrum (and in Vogelaar's data), comparison to the "template" spectra was limited to the range 2.0 to 5.5 MeV.

An additional complication of the Vogelaar experiment was a background due to 11.67 and 4.44 MeV gamma rays from the ^{11}B(p,γ)^{12}C reaction. Vogelaar accounted for this by subtracting background obtained in a run at an energy below the ^{26g}Al resonance. In the GEANT4 simulation 10 runs of 20000 events, having about the same amount of background as seen in Fig. 5.9, were generated. To mimic the procedure of Vogelaar, a spectrum formed by the difference between a pair of background histograms was added to the simulated ^{26}Al(p,γ)^{27}Si spectrum.

The PHYSICA[5] analysis package was used for fitting of the spectra by χ^2 minimization as well as plotting of spectra. The weighting factor for fitting was taken to be the inverse of the expected variance in counts for each bin. To estimate the background variance the 10 background runs were summed and divided by 5. The variance in ^{27}Si data was calculated from the "theoretical" spectra (Figs. 4 to 8) weighted by the best-fit normalization factor and the "true" branching ratios. Fig. 9 shows a typical result for a fit of the "4.45" and "2.91" templates to a "V-type" 800-event simulation.

In a first set of ten 800-event runs, spectra were generated according to the "V" class decay scheme proposed by Vogelaar[1]. Each was fitted to the four types of allowed transition pairs (see above). The run-by-run results for the best-fit branching ratios and their 1σ uncertainties are plotted in Figure 10.

A second set of ten 800-event runs was generated according to the "L" class, with results as shown in Figure 11. The 10 ^{11}B(p,γ) spectra used to calculate backgrounds for the "V" runs were reused here, with the same sequence of pairings. A third set of

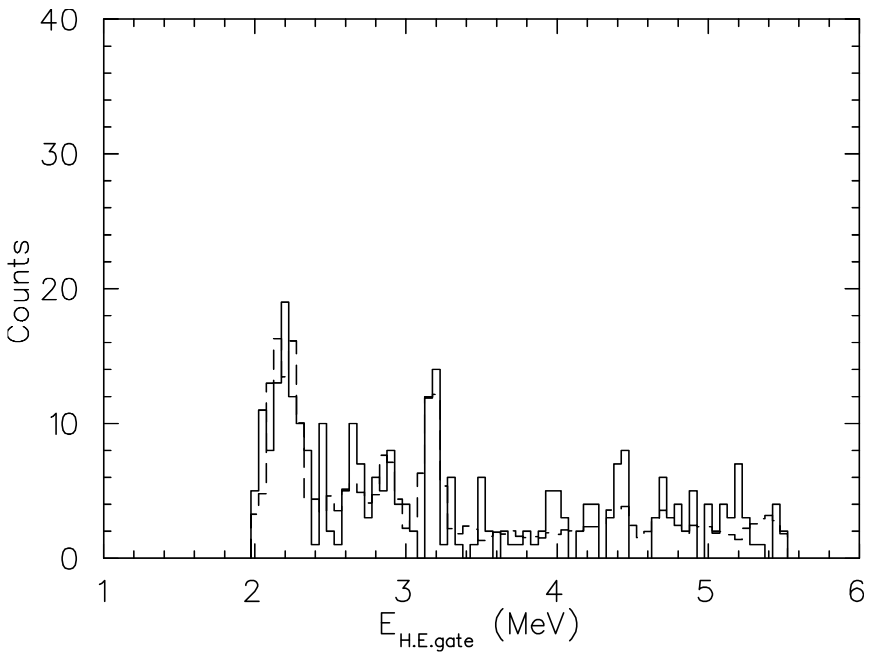

FIGURE 9. Fitting of GEANT4-generated "data" to template spectra generated by high-statistics GEANT4 runs. The solid line is the low-statistics "data" and the broken line the result of the fit.

ten 800-event runs was based on the "7+7" branching scheme, with results as shown in Fig. 12.

Not surprisingly, the fits for a 2.91 MeV branch were 0.10 or greater in 6 out of the 10 runs of "V" type, with 1σ uncertainty of about 0.06 and an average of 0.093 over 10 runs. For the 10 "L" runs the 2.91 MeV branch never had a fit value 0.10 or more and averaged 0.002. More surprising is the fact that the 2.91 MeV branch had a average fit value -0.02 and only 1 run with a value as large as 0.10 for the "7+7" set: recall that the "7+7" and Vogelaar's decay scheme have almost the same number of 2.91 MeV gamma rays per 1000 decays of the 7.65 MeV state. A possible explanation is that the decay directly to the 2.91 MeV state is predominantly a 2-gamma cascade, while all of the constituents of the "7+7" runs are cascades of 3 (or more) gamma rays (see template spectra, above). For the large NaI detectors, fitting is affected by the entire spectrum, not just a full-energy peak.

3.2. **4- and 5-parameter fits to branching ratios.** A reasonable next step is to do a fit allowing the branching ratios to the 4.45 MeV state plus the states decaying to the 2.91 MeV state to vary simultaneously. This was done for the same sets of ten 800-event runs as before. Free variables were an overall normalization factor plus branching ratios for the 2.91, 5.26, 5.28 and 5.55 MeV levels. Figure 13 shows the 5-parameter fit for run 5 of the "7+7" set. It was observed that the assigned uncertainties for the branches to the 5.26 and 5.28 MeV states were very large — approximately 0.2. This was found to be due to a very high degree of correlation (-0.87) between those two branching ratios. Subsequent multi-parameter fits were made with the 5.26 MeV

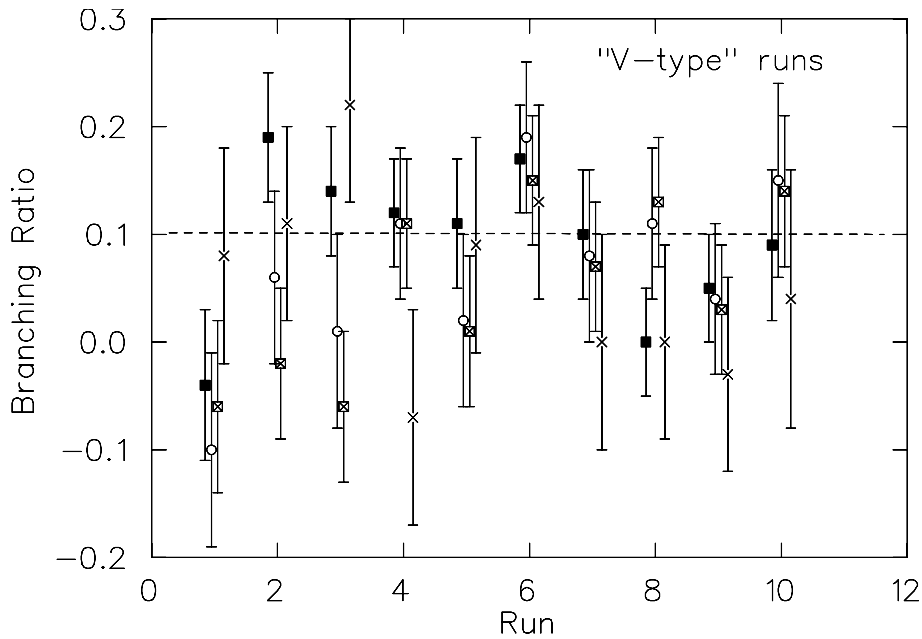

FIGURE 10. Best fits to simulated runs of "V-type" assuming decay to the 4.45 MeV state and one other state. The resulting branching ratios and 1σ error bars are shown for the second state being at 2.91 MeV (solid squares), 5.26 MeV (cross in box), 5.28 MeV (open circles) or 5.55 MeV (cross).

branch set to zero (but recognizing that the sensitivity is to the *sum* of branches to the 5.26 and 5.28 MeV states).

Four-parameter fits were made for "V" type and "7+7" type runs. Results appear in Figures 14 and 15. Again, as with the 2-parameter fits described in the previous sub-section, the fit values for the 7.65→2.91 branch are markedly different between the "V" and "7+7" classes of 800-event runs. Of the 10 "7+7" runs, there is only 1 run for which the 2.91 branching ratio was positive by more than 1σ (and in that run the 5.28 and 5.55 branches had larger, positive values). In contrast, for the "V" set of runs, the 2.91 branch was positive by more than 1σ in 6 of the 10 runs, was greater than 0.10 for 5 of these and was the only branching ratio positive by more than 1σ for 3 runs.

Some of these fits gave unphysical results: branching ratios with negative values. The 3 branching ratios had negative correlations (of order -0.35 to -0.2), so setting a negative branching ratio to zero would have the effect of "pulling down" the values for the other branches. When runs 2–5 of Fig. 14 were re-calculated with negative branching ratios fixed at zero, the branching ratios for the 2.91 MeV branch indeed became less positive by small amounts, but only in the case of run 5 was it brought from above 0.10 to below (to 0.097).

An interesting feature of the fits is that the uncertainty in branching ratio is substantially smaller (typically 0.06 compared to 0.09) for the 2.91 branch than for the 5.28 or 5.55 branches. The spectrum for the 2.91 MeV branch is, in some sense, "more orthogonal" to the spectrum of the dominant 7.65→4.45 branch.

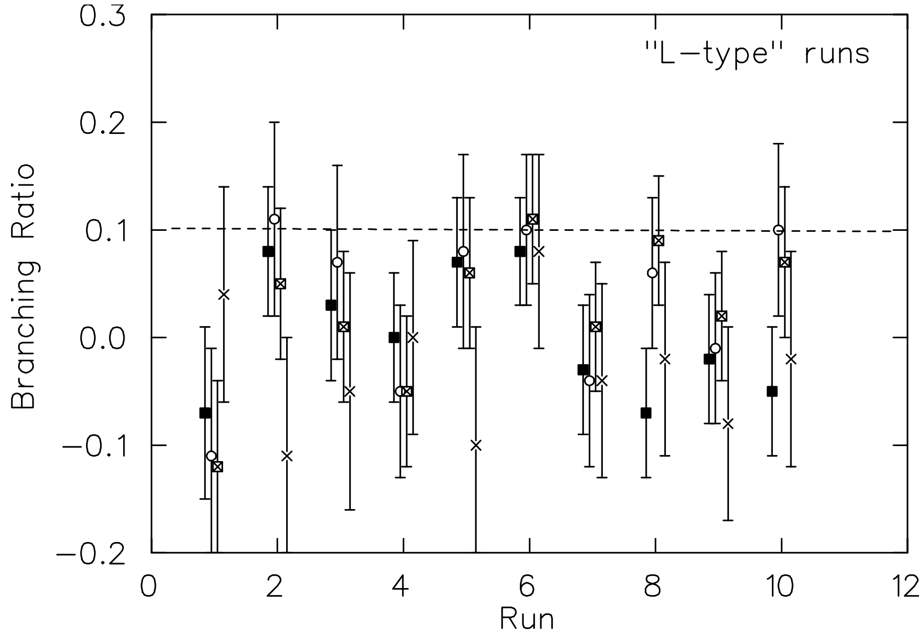

FIGURE 11. Best fits to simulated runs of "L-type" assuming decay to the 4.45 MeV state and one other state. The symbols have the same meaning as in Fig. 10.

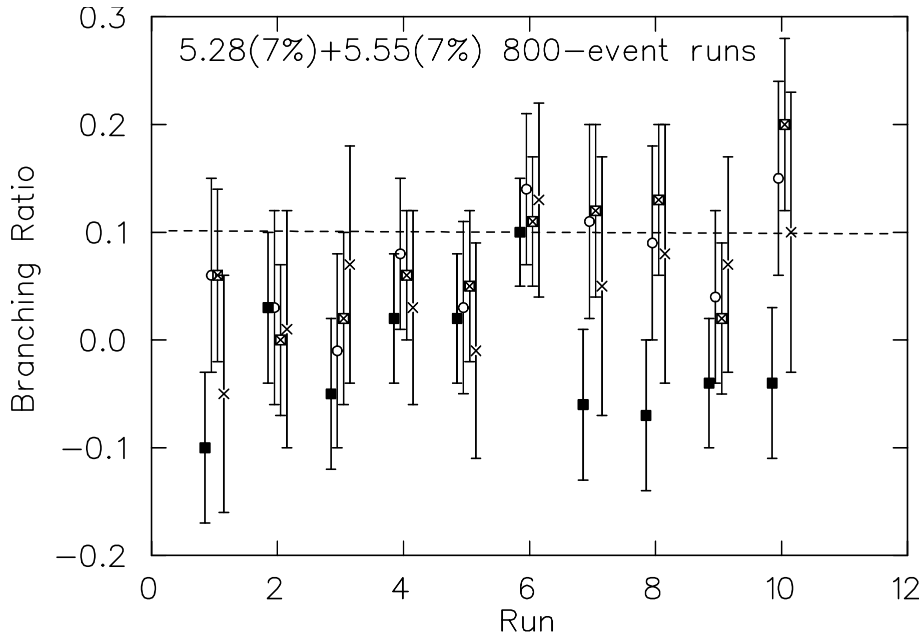

FIGURE 12. Best fits to simulated runs of "7+7" assuming decay to the 4.45 MeV state and one other state. The symbols have the same meaning as in Fig. 10.

4. Discussion

This investigation has attempted to follow lines which might reasonably have been taken by Vogelaar[1], knowing that he relied on extensive EGS simulation and coincidence gating to identify the principal cascade through the 4.45 MeV state. A plausible

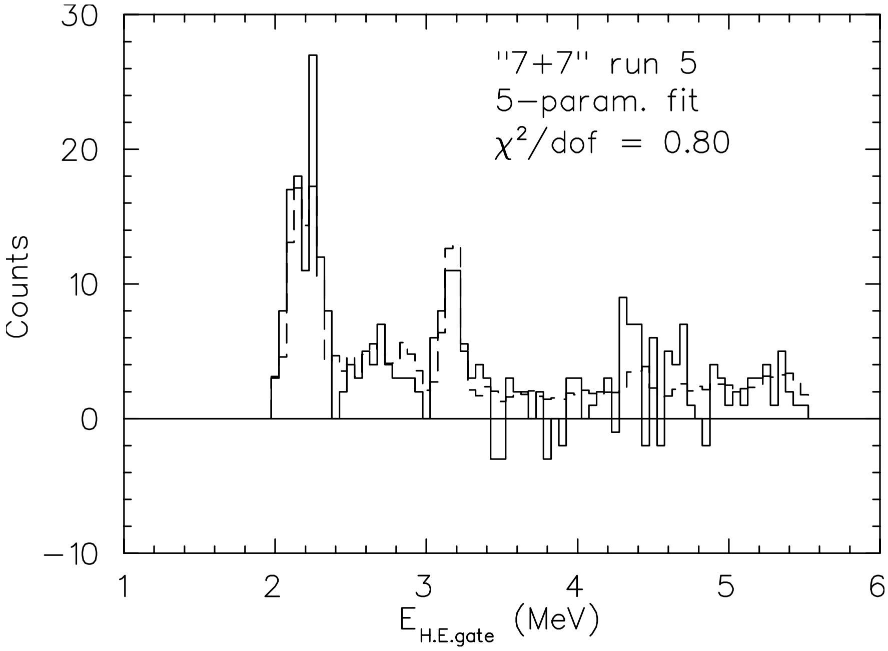

FIGURE 13. Fit to simulation run 5 of "7+7" type, allowing variation of overall intensity and branches to states at 2.91, 5.26, 5.28 and 5.55 MeV.

analysis protocol has come to the conclusion that 1-step and 2-step branches can have quite different fingerprints, even when they populate the 2.91 MeV state by the same amount. There is a substantial probability — of order 3 out of 10 — that analysis of a one-off experiment would find a clear preference for Vogelaar's decay scheme if the true decay mode were 10% to the 2.91 MeV state with no 2-step feeding. Two decay schemes based on 2-step decays to the 2.91 MeV state did not produce an instance (in 10 trials each) where the 1-step branch was the clearly-preferred result of a fit.

Any conclusion about the existence or non-existence of a doublet of states at 7.65 MeV must be tempered by a number of caveats:

- *the simulation may not be a sufficiently accurate description of the experiment.* The main effect of adding material is to remove some events from the 7.2–7.75 MeV cut on summed energy: spectrum shape was hardly affected. The GEANT4 simulation produced a spectrum very similar to that from EGS4 for the dominant cascade. Vogelaar would have known the exact dimensions of experimental equipment and could have made the EGS4 simulation to match; the GEANT4 runs by construction were a perfect description of the simulated experiment.
- *only 3 possible decay scenarios were considered, none of them necessarily what was observed in the fusion-evaporation experiment* This can be remedied when numerical results for branching ratios in the Gammasphere experiment become publicly available.

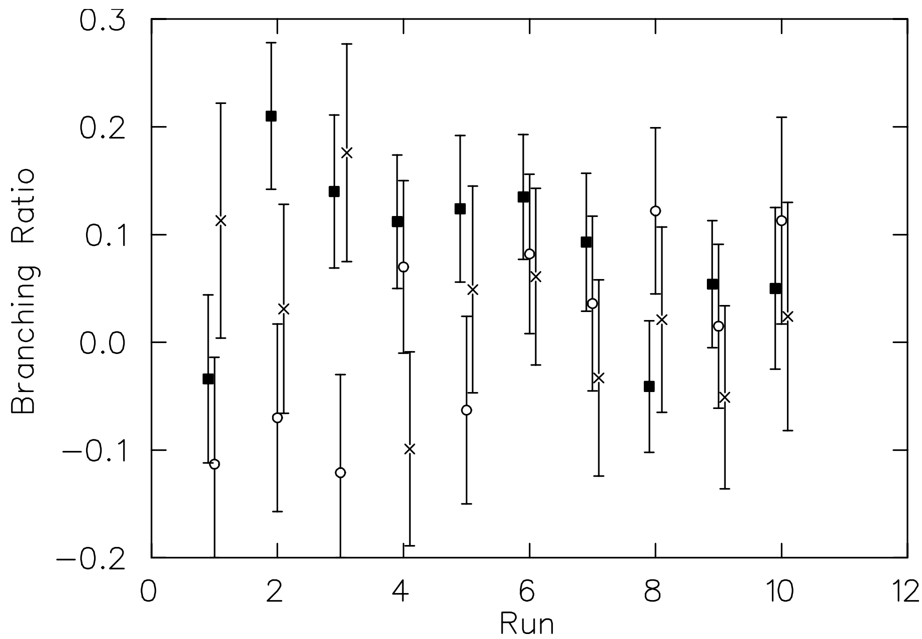

FIGURE 14. Best fits to simulated runs of 'V" type runs, with simultaneous variation of branches to the 2.91, 5.82 and 5.55 MeV states plus an overall normalization. The symbols have the same meaning as in Fig. 10.

- *bin size and energy resolution may not be exactly correct.* Bin size is important if in the original experiment it was large enough to worsen the effective energy resolution. For a 2.25 MeV gamma ray the detector photo-peak resolution is estimated to be σ=45 keV to be compared with the binning contribution of $50/\sqrt{12}$ keV. Energy resolution was adjusted to match that of Vogelaar's EGS4 simulation in the key region between 2 and 2.5 MeV.
- *the 4.74 MeV peak isn't seen clearly in the simulated "Vogelaar type" spectra.* Obviously, it is more convincing to see peaks in an energy spectrum than to believe results of a fit to broad structures over a span of 3 MeV. However, provided due care has been taken to check the correctness of the model, lack of a visible peak does not constitute grounds for rejecting the results of a fit.

5. SUMMARY

GEANT4 simulation has shown that, despite limitations from background, low statistics and moderate energy resolution, it may have been possible for Vogelaar to have distinguished between possible minor (5–15%) branches of the 7.65 MeV state. In particular, there seems to be sensitivity to whether decay to the 2.91 MeV level takes place by 1-step *vs* 2-step transitions. For the particular cases considered, there was about a 3-in-10 chance of correctly identifying a 1-step branch of 10%.

Any conclusion stronger than "may have been possible" is made difficult by ignorance of details of Vogelaar's analysis method and of the precise values for branching ratios from the Lotay *et al.* experiment.

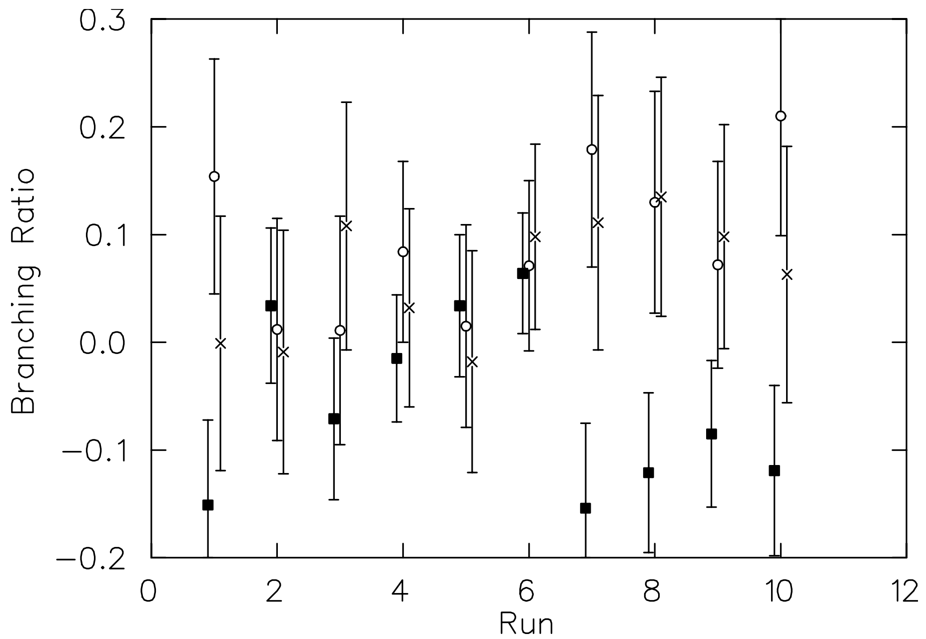

FIGURE 15. Best fits to simulated runs of '7+7" type runs, with simultaneous variation of branches to the 2.91, 5.82 and 5.55 MeV states plus an overall normalization. The symbols have the same meaning as in Fig. 10.

References

[1] R.B.Vogelaar, Ph.D. thesis Cal.Tech. (1989).
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